



FIGURE 5.10

The amplitude and phase of two reflection coefficients are shown as a function of p , the horizontal slowness. The coefficients are $\hat{S}\hat{P}$ and $\hat{S}\hat{S}$ for an SV -wave incident on the free surface, and we have taken $\alpha = 5$ km/s, $\beta = 3$ km/s, so that these coefficients have already been shown in Figure 5.6 for the range $0 \leq p \leq 1/\alpha$. Here we extend the range to $0 \leq p \leq 1/\beta$, so that an inhomogeneous P -wave is present for the range $1/\alpha < p \leq 1/\beta$. We have chosen to emphasize phase advance rather than phase shift, since the former is independent of the sign of frequency and independent of our Fourier sign convention. The phases actually plotted are those of $\hat{S}\hat{P}$ and $\hat{S}\hat{S}$ as determined by (5.31) and (5.32). Note that zeros in the coefficients are now associated with jumps of amount π in phase.

$1/\alpha < 1/\beta < p$. In this case, we have a very different overall picture from that considered so far, since now the energy is no longer transmitted toward the boundary and scattered from it. Rather, it can be channelled only along the boundary itself. For a half-space we cannot permit unbounded waves, so that the only permissible wave types are those which exponentially decay with distance from the surface:

$$\hat{P} \left(\alpha p, 0, i\sqrt{\alpha^2 p^2 - 1} \right) \exp \left(-\omega \sqrt{p^2 - \frac{1}{\alpha^2}} z \right) \exp[i\omega(px - t)] \quad (5.52)$$

for the inhomogeneous P -wave, and

$$\hat{S} \left(i\sqrt{\beta^2 p^2 - 1}, 0, -\beta p \right) \exp \left(-\omega \sqrt{p^2 - \frac{1}{\beta^2}} z \right) \exp[i\omega(px - t)] \quad (5.53)$$